Tables of the Generating Functions and Groundforms for Simultaneous Binary Quantics of the First Four Orders, taken two and two together.

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In the Generating Functions given below, the exponents of the letters a, b, c, d, refer to degree in the coefficients of the quantics of the 1st, 2nd, 3rd and 4th orders respectively; the exponents of the letter x to order in the variables. Where the system consists of two quantics of the same order, the Latin letter and the corresponding Greek letter have been used. In the tabulated numerators, the *minus* sign has been placed *over* the number which it affects.

In each of the systems considered in this paper, with the exception of that consisting of a cubic and a quartic, it is found that there is never more than one groundform of any given type (*i. e.* of a given order in the variables and given degrees in the coefficients of the quantics); where, therefore, in the enumeration of the groundforms, the *type* alone is given, the *number* of groundforms of the type is to be understood to be 1. The symbol (λ, μ) is used to indicate a form of the degrees λ and μ in the coefficients of the two quantics, the number placed first always relating to the quantic of lower order, when the orders are different. In the last three cases, the numbers, as well as the types, of the groundforms are given in tables, which require no explanation.

System of Two Linears.*

G. F. for differentiants, $\dfrac{1}{(1-a)(1-\alpha)(1-a\alpha)}$.

G. F. for covariants, $\dfrac{1}{(1-a\alpha)(1-ax)(1-\alpha x)}$.

Groundforms:

Of order 0 . . . . (1, 1).

" " 1 . . . . (0, 1), (1, 0).

* "Linear" is here used as a noun, in conformity with the use of the words quadric, cubic, &c.

SYSTEM OF LINEAR AND QUADRIC.

G. F. for differentiants, $\dfrac{1+ab}{(1-a)(1-b)(1-b^2)(1-a^2b)}$.

G. F. for covariants, $\dfrac{1+abx}{(1-b^2)(1-a^2b)(1-ax)(1-bx^2)}$.

Groundforms:

Of order 0 . . . . (0, 2), (2, 1).
" " 1 . . . . (1, 0), (1, 1).
" " 2 . . . . (0, 1).

SYSTEM OF LINEAR AND CUBIC.

G. F. for differentiants, $\dfrac{1+a^2c+(a-a^3)c^2+(1-a^2)c^3-ac^4-a^3c^5}{(1-a)(1-c)(1-c^2)(1-c^4)(1-ac)(1-a^3c)}$.

G. F. for covariants, reduced form,

Denominator: $(1-c^4)(1-ac)(1-a^3c)(1-ax)(1-cx)(1-cx^3)$.

Numerator: $1-ac+a^2c^2+\{(-1+a^2)c+(2a-a^3)c^2-a^2c^3\}x$
$+\{ac+(1-2a^2)c^2+(-a+a^3)c^3\}x^2+\{-ac^2+a^2c^3-a^3c^4\}x^3$.

G. F. for covariants, representative form,

Denominator: $(1-c^4)(1-a^3c)(1-a^2c^2)(1-ax)(1-c^2x^2)(1-cx^3)$.

Numerator: $1+a^3c^3+\{a^2c+ac^2+(a^2-a^4)c^3\}x+\{ac+(a-a^3)c^3-a^3c^5\}x^2$
$+\{(1-a^2)c^3-a^3c^4-a^2c^5\}x^3+\{-ac^3-a^4c^6\}x^4$.

Groundforms:

Of order 0 . . . . (0, 4), (2, 2), (3, 1), (3, 3).
" " 1 . . . . (1, 0), (1, 2), (2, 1), (2, 3).
" " 2 . . . . (0, 2), (1, 1), (1, 3).
" " 3 . . . . (0, 1), (0, 3).

SYSTEM OF LINEAR AND QUARTIC.

G. F. for differentiants, $\dfrac{1+(a+a^3)d+(a+a^2-a^5)d^2+(1-a^3-a^4)d^3+(-a^2-a^4)d^4-a^5d^5}{(1-a)(1-d)(1-d^2)^2(1-d^3)(1-a^2d)(1-a^4d)}$.

G. F. for covariants, reduced form,

Denominator: $(1-d^2)(1-d^3)(1-a^2d)(1-a^4d)(1-ax)(1-dx^2)(1-dx^4)$.

Numerator: $1-a^2d+a^4d^2+\{a^3d+(a^3-a^5)d^2\}x+\{(-1+a^2)d$
$+(2a^2-a^4)d^2-a^4d^3\}x^2+\{ad+(a-2a^3)d^2+(-a^3+a^5)d^3\}x^3$
$+\{(1-a^2)d^2-a^2d^3\}x^4+\{-ad^2+a^3d^3-a^5d^4\}x^5$.

G. F. for covariants, representative form,

Denominator: $(1-d^2)(1-d^3)(1-a^4d)(1-a^4d^2)(1-ax)(1-dx^4)(1-d^2x^4)$.

Numerator: $1+a^6d^3+\{a^3d+a^3d^2+(a^5-a^7)d^3\}x+\{a^2d+a^2d^2+(a^4-a^6)d^3\}x^2$
$+\{ad+ad^2+(a^3-a^5)d^3\}x^3+\{(a^2-a^4)d^3-a^6d^4-a^6d^5\}x^4$
$+\{(a-a^3)d^3-a^5d^4-a^5d^5\}x^5+\{(1-a^2)d^3-a^4d^4-a^4d^5\}x^6$
$+\{-ad^3-a^7d^6\}x^7$.

Groundforms:

Of order 0	(0, 2), (0, 3), (4, 1), (4, 2), (6, 3).
" " 1	(1, 0), (3, 1), (3, 2), (5, 3).
" " 2	(2, 1), (2, 2), (4, 3).
" " 3	(1, 1), (1, 2), (3, 3).
" " 4	(0, 1), (0, 2), (2, 3).
" " 5	(1, 3).
" " 6	(0, 3).

System of Two Quadrics.

G. F. for differentiants, $\dfrac{1+b\beta}{(1-b)(1-b^2)(1-\beta)(1-\beta^2)(1-\beta b)}$.

G. F. for covariants, $\dfrac{1+b\beta x^2}{(1-b^2)(1-\beta^2)(1-b\beta)(1-bx^2)(1-\beta x^2)}$.

Groundforms:

Of order 0	(0, 2), (1, 1), (2, 0).
" " 2	(0, 1), (1, 0), (1, 1).

System of Quadric and Cubic.

G. F. for differentiants,

$$\frac{1+(2b+b^2)c+(b+b^2+b^3)c^2+c^3-b^4c^4+(-b-b^2-b^3)c^5+(-b^2-2b^3)c^6-b^4c^7}{(1-b)(1-b^2)(1-c)(1-c^2)(1-c^4)(1-bc^2)(1-b^3c^2)}.$$

G. F. for covariants, reduced form,

Denominator: $(1-b^2)(1-c^4)(1-bc^2)(1-b^3c^2)(1-bx^2)(1-cx)(1-cx^3)$.

Numerator: $1+b^3c^4+\{(-1+b+b^2)c+(b+b^2)c^3-b^3c^5\}x$
$+\{(1+b^3)c^2+(-b-b^4)c^4\}x^2+\{bc+(-b^2-b^3)c^3$
$+(-b^2-b^3+b^4)c^5\}x^3+\{-bc^2-b^4c^6\}x^4$.

G. F. for covariants, representative form,

Denominator: $(1-b^2)(1-c^4)(1-bc^2)(1-b^3c^2)(1-bx^2)(1-c^2x^2)(1-cx^3)$.

Numerator: $1+b^3c^4+\{(b+b^2)c+(b+b^2)c^3\}x+\{(b+b^2+b^3)c^2$
$+(b^2-b^4)c^4-b^3c^6\}x^2+\{bc+(1-b^2)c^3+(-b-b^2-b^3)c^5\}x^3$
$+\{(-b^2-b^3)c^4+(-b^2-b^3)c^6\}x^4+\{-bc^3-b^4c^7\}x^5$.

Groundforms:

Of order 0 . . . . (0, 4), (1, 2), (2, 0), (3, 2), (3, 4)
" " 1 . . . . (1, 1), (1, 3), (2, 1), (2, 3).
" " 2 . . . . (0, 2), (1, 0), (1, 2).
" " 3 . . . . (0, 1), (0, 3), (1, 1).

SYSTEM OF QUADRIC AND QUARTIC.

G. F. for differentiants, $\dfrac{1+(b+b^2)\,d+(2b-b^3)\,d^2+(1-2b^2)\,d^3+(-b-b^2)\,d^4-b^3d^5}{(1-b)(1-b^2)(1-d)(1-d^2)^2(1-d^3)(1-bd)(1-b^2d)}$.

G. F. for covariants, reduced form,

Denominator: $(1-b^2)(1-d^2)(1-d^3)(1-bd)(1-b^2d)(1-bx^2)(1-dx^2)$ $(1-dx^4)$.

Numerator: $1-bd+b^2d^2+\{(-1+b+b^2)\,d+(2b-b^3)\,d^2-b^2d^3\}\,x^2$ $+\{bd+(1-2b^2)\,d^2+(-b-b^2+b^3)\,d^3\}\,x^4+\{-bd^2$ $+b^2d^3-b^3d^4\}\,x^6$.

G. F. for covariants, representative form,

Denominator: $(1-b^2)(1-d^2)(1-d^3)(1-b^2d)(1-b^2d^2)(1-bx^2)(1-dx^4)$ $(1-d^2x^4)$.

Numerator: $1+b^3d^3+\{(b+b^2)\,d+(b+b^2)\,d^2+(b^2-b^4)\,d^3\}\,x^2+\{bd+bd^2$ $+(b-b^3)\,d^3-b^3d^4-b^3d^5\}\,x^4+\{(1-b^2)\,d^3+(-b^2-b^3)\,d^4$ $+(-b^2-b^3)\,d^5\}\,x^6+\{-bd^3-b^4d^6\}\,x^8$.

Groundforms:

Of order 0 . . . . (0, 2), (0, 3), (2, 0), (2, 1), (2, 2), (3, 3).
" " 2 . . . . (1, 0), (1, 1), (1, 2), (2, 1), (2, 2), (2, 3).
" " 4 . . . . (0, 1), (0, 2), (1, 1), (1, 2), (1, 3).
" " 6 . . . . (0, 3).

SYSTEM OF TWO CUBICS.

G. F. for differentiants,

Denominator: $(1-c)(1-c^2)(1-c^4)(1-\gamma)(1-\gamma^2)(1-\gamma^4)(1-c\gamma)$ $(1-c^3\gamma)(1-c\gamma^3)$.

Numerator: $1+c^3+(2c+2c^2-c^5-c^6)\,\gamma+(2c+2c^2-c^4-c^5-c^6-c^7)\,\gamma^2$ $+(1+2c^3-c^4-2c^5-c^6-c^7)\,\gamma^3+(-c^2-c^3-c^5-c^6)\,\gamma^4$ $+(-c-c^2-2c^3-c^4+2c^5+c^8)\gamma^5+(-c-c^2-c^3-c^4+2c^6+2c^7)\gamma^6$ $+(-c^2-c^3+2c^6+2c^7)\,\gamma^7+(c^5+c^8)\,\gamma^8$.

G. F. for covariants, reduced form,

Denominator: $(1-c^4)(1-\gamma^4)(1-c\gamma)(1-c^3\gamma)(1-c\gamma^3)(1-cx)(1-cx^3)$
$(1-\gamma x)(1-\gamma x^3)$

Numerator:

		γ^0	γ^1	γ^2	γ^3	γ^4	γ^5	γ^6
x^0	c^0	1						
	c^2			1				
	c^3				1			
	c^5						1	
x^1	c^0		$\bar{1}$					
	c^1	$\bar{1}$		1		1		
	c^2		1					
	c^4		1					
	c^5							$\bar{1}$
	c^6						$\bar{1}$	
x^2	c^0			1				
	c^1		2				$\bar{1}$	
	c^2	1		$\bar{1}$		$\bar{1}$		
	c^4			$\bar{1}$				
	c^5		$\bar{1}$					
	c^6							1

		γ^1	γ^2	γ^3	γ^4	γ^5	γ^6	γ^7
x^6	c^2		1					
	c^4				1			
	c^5					1		
	c^7							1
x^5	c^1		$\bar{1}$					
	c^2	$\bar{1}$						
	c^3						1	
	c^5						1	
	c^6			1		1		$\bar{1}$
	c^7						$\bar{1}$	
x^4	c^1	1						
	c^2						$\bar{1}$	
	c^3					$\bar{1}$		
	c^5			$\bar{1}$		$\bar{1}$		1
	c^6		$\bar{1}$				2	
	c^7					1		

		γ^1	γ^2	γ^3	γ^4	γ^5	γ^6
x^3	c^1		$\bar{1}$		$\bar{1}$		
	c^2	$\bar{1}$				1	
	c^3				$\bar{1}$		$\bar{1}$
	c^4	$\bar{1}$		$\bar{1}$			
	c^5		1				$\bar{1}$
	c^6			$\bar{1}$		$\bar{1}$	

G. F. for covariants, representative form,

Denominator: $(1-c^4)(1-\gamma^4)(1-c\gamma)(1-c^3\gamma)(1-c\gamma^3)(1-c^2x^2)(1-cx^3)$
$(1-\gamma^2x^2)(1-\gamma x^3)$.

Numerator:

		γ^0	γ^1	γ^2	γ^3	γ^4	γ^5	γ^6	γ^7
x^0	c^0	1							
	c^2			1					
	c^3				1				
	c^5						1		
x^1	c^1			1		1			
	c^2		1		1				
	c^3			1		1			
	c^4		1		1				
x^2	c^1		1		1				
	c^2			1					
	c^3		1		1				
	c^4					1			
	c^5								$\bar{1}$
	c^7						$\bar{1}$		
x^3	c^0				1				
	c^1			1		$\bar{1}$		$\bar{1}$	
	c^2		1						
	c^3	1				$\bar{2}$		$\bar{1}$	
	c^4		$\bar{1}$		$\bar{2}$				
	c^5							$\bar{1}$	
	c^6		$\bar{1}$		$\bar{1}$		$\bar{1}$		

		γ^1	γ^2	γ^3	γ^4	γ^5	γ^6	γ^7	γ^8
x^8	c^3			1					
	c^5					1			
	c^6						1		
	c^8								1
x^7	c^4					1		1	
	c^5				1		1		
	c^6					1		1	
	c^7				1		1		
x^6	c^1			$\bar{1}$					
	c^3	$\bar{1}$							
	c^4				1				
	c^5					1		1	
	c^6						1		
	c^7					1		1	
x^5	c^2			$\bar{1}$		$\bar{1}$		$\bar{1}$	
	c^3		$\bar{1}$						
	c^4					$\bar{2}$		$\bar{1}$	
	c^5		$\bar{1}$		$\bar{2}$				1
	c^6							1	
	c^7		$\bar{1}$		$\bar{1}$		1		
	c^8					1			

		γ^1	γ^2	γ^3	γ^4	γ^5	γ^6	γ^7
x^4	c^1	1				$\bar{1}$		
	c^2				$\bar{1}$		$\bar{1}$	
	c^3			$\bar{1}$		$\bar{2}$		$\bar{1}$
	c^4		$\bar{1}$		$\bar{2}$		$\bar{1}$	
	c^5	$\bar{1}$		$\bar{2}$		$\bar{1}$		
	c^6		$\bar{1}$		$\bar{1}$			
	c^7			$\bar{1}$				1

*Table of Groundforms.**

Order in the Variables.	Deg. in coeff's of 2d cubic.	Deg. in coeff's of 1st cubic. 0	1	2	3	4
0	0					1
	1		1		1	
	2			1		
	3		1		1	
	4	1				
1	1			1		1
	2		1		1	
	3			1		
	4		1			

Order in the Variables.	Deg. in coeff's of 2d cubic.	Deg. in coeff's of 1st cubic. 0	1	2	3
2	0			1	
	1		1		1
	2	1		1	
	3		1		
3	0		1		1
	1	1		1	
	2		1		
	3	1			
4	1		1		

System of Cubic and Quartic.

G. F. for differentiants,

Denominator: $(1-c)(1-c^2)(1-c^4)(1-d)(1-d^2)^2(1-d^3)(1-c^2d)$
$(1-c^4d)(1-c^2d^3)(1-c^4d^3)$.

Numerator: $1+c^3+(3c+2c^2+2c^3+c^4-2c^5-c^6-c^7)\,d$
$+(3c+5c^2+2c^3+2c^4-3c^5-4c^6-2c^7-2c^8+c^9)\,d^2$
$+(1+3c^2+3c^3+c^4-c^5-6c^6-5c^7-4c^8+2c^{10})\,d^3$
$+(-c^2+c^3-c^4-2c^5-5c^6-6c^7-3c^8-c^9+3c^{10}+2c^{11}+c^{12})\,d^4$
$+(-2c^2-3c^3-3c^4-3c^5-2c^6-2c^7-c^8+2c^{10}+4c^{11}+3c^{12}+c^{13})\,d^5$
$+(-c^2-3c^3-4c^4-2c^5+c^7+2c^8+2c^9+3c^{10}+3c^{11}+3c^{12}+2c^{13})\,d^6$
$+(-c^3-2c^4-3c^5+c^6+3c^7+6c^8+5c^9+2c^{10}+c^{11}-c^{12}+c^{13})\,d^7$
$+(-2c^5+4c^7+5c^8+6c^9+c^{10}-c^{11}-3c^{12}-3c^{13}-c^{15})\,d^8$
$+(-c^6+2c^7+2c^8+4c^9+3c^{10}-2c^{11}-2c^{12}-5c^{13}-3c^{14})\,d^9$
$+(c^8+c^9+2c^{10}-c^{11}-2c^{12}-2c^{13}-3c^{14})\,d^{10}+(-c^{12}-c^{15})\,d^{11}$.

G. F. for covariants, reduced form,

Denominator: $(1-c^4)(1-d^2)(1-d^3)(1-c^2d)(1-c^4d)(1-c^2d^3)(1-c^4d^3)$
$(1-cx)(1-cx^3)(1-dx^2)(1-dx^4)$.

*The forms of ord. 1, deg. 3, 4 and of ord. 1, deg. 4, 3 given by Clebsch and Gordan, do not appear in this table, and it has been proved by the author that no fundamental forms of either of these types exist.

Numerator:

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9
x^0	c^0	1									
	c^2		$\bar{1}$								
	c^4			2	2	2	1				
	c^6			1	1		$\bar{1}$	$\bar{1}$			
	c^8				$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{2}$			
	c^{10}								1		
	c^{12}									$\bar{1}$	
x^1	c^1	$\bar{1}$		1							
	c^3		3	2	1	1					
	c^5		1	$\bar{2}$	$\bar{1}$	$\bar{1}$	$\bar{1}$	1			
	c^7			$\bar{2}$	$\bar{1}$	$\bar{1}$		1	$\bar{1}$		
	c^9				1	1	1		$\bar{2}$		
	c^{11}							$\bar{1}$	$\bar{2}$		
	c^{13}									1	
x^2	c^0		$\bar{1}$								
	c^2	1	1	3	2	1					
	c^4		$\bar{1}$		$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{1}$			
	c^6		$\bar{1}$		$\bar{2}$	$\bar{2}$			1		
	c^8			1		1	1		2		
	c^{10}						$\bar{1}$	$\bar{1}$		$\bar{2}$	
	c^{12}							1	1		1
x^3	c^1		2								
	c^3			$\bar{3}$							
	c^5		$\bar{1}$	$\bar{2}$	1		1		$\bar{1}$		
	c^7					$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	1	
	c^9						$\bar{1}$			1	
	c^{11}					1	1	2	1	2	
	c^{13}									1	$\bar{1}$

		d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}
x^8	c^2		1								
	c^4			$\bar{1}$							
	c^6				2	2	2	1			
	c^8				1	1		$\bar{1}$	$\bar{1}$		
	c^{10}.					$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{2}$		
	c^{12}									1	
	c^{14}										$\bar{1}$
x^7	c^1		$\bar{1}$								
	c^3			2	1						
	c^5			2		$\bar{1}$	$\bar{1}$	$\bar{1}$			
	c^7			1	$\bar{1}$		1	1	2		
	c^9				$\bar{1}$	1	1	1	2	$\bar{1}$	
	c^{11}						$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	
	c^{13}								$\bar{1}$	$\bar{1}$	1
x^6	c^2	$\bar{1}$		$\bar{1}$	$\bar{1}$						
	c^4		2		1	1					
	c^6			$\bar{2}$		$\bar{1}$	$\bar{1}$		$\bar{1}$		
	c^8			$\bar{1}$			2	2		1	
	c^{10}				1	2	2	1		1	
	c^{12}						$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$	$\bar{1}$
	c^{14}									1	
x^5	c^1	1.	$\bar{1}$								
	c^3		$\bar{2}$	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{1}$				
	c^5		$\bar{1}$			1					
	c^7		$\bar{1}$	1	2	1	1				
	c^9			1		$\bar{1}$		$\bar{1}$	2	1	
	c^{11}								3		
	c^{13}									$\bar{2}$	

Numerator—*Continued:*

		d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9
x^4	c^0		1							
	c^2	$\bar{1}$		$\bar{1}$	$\bar{1}$	$\bar{1}$				
	c^4	$\bar{1}$		$\bar{2}$	$\bar{2}$	$\bar{1}$				
	c^6		1	$\bar{2}$	$\bar{1}$			1		
	c^8			$\bar{1}$			1	2	$\bar{1}$	
	c^{10}					1	2	2		1
	c^{12}					1	1	1		1
	c^{14}								$\bar{1}$	

G. F. for covariants, representative form,

Denominator: $(1-c^4)(1-d^2)(1-d^3)(1-c^4d)(1-c^4d^2)(1-c^2d^3)(1-c^4d^3)$
$(1-cx^3)(1-c^2x^2)(1-dx^4)(1-d^2x^4)$.

Numerator:

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}
x^0	c^0	1											
	c^4			1	2	2	1						
	c^6			1	3	2	1						
	c^8					$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$				
	c^{10}					$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{1}$				
	c^{14}										$\bar{1}$		
x^1	c^1		1	1									
	c^3		2	3	2	1							
	c^5		1	2	3	2	1	1					
	c^9				$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$	$\bar{1}$			
	c^{11}						$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$			
	c^{13}								$\bar{1}$	$\bar{1}$			

		d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}	d^{12}
x^{11}	c^3			1									
	c^7					1	2	2	1				
	c^9					1	3	2	1				
	c^{11}							$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$		
	c^{13}							$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{1}$		
	c^{17}												$\bar{1}$
x^{10}	c^4				1	1							
	c^6				2	3	2	1					
	c^8				1	2	3	2	1	1			
	c^{12}						$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	
	c^{14}								$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$	
	c^{16}										$\bar{1}$	$\bar{1}$	

Numerator—*Continued:*

		d^0	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}
x^2	c^2		2	3	2	1							
	c^4		2	4	5	3	1						
	c^8			$\bar{1}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{2}$	$\bar{1}$				
	c^{10}						$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$			
	c^{12}					1	1		$\bar{2}$	$\bar{2}$	$\bar{1}$		
	c^{16}										1		
x^3	c^1		1	1	1								
	c^3	1	1	3	5	3	1						
	c^5		$\bar{1}$		2	3	2						
	c^7		$\bar{1}$	$\bar{2}$	$\bar{4}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{1}$				
	c^9					$\bar{2}$	$\bar{4}$	$\bar{3}$	$\bar{3}$	$\bar{1}$			
	c^{11}				1	1	1		$\bar{1}$	$\bar{1}$	$\bar{1}$		
	c^{13}						1	1	2	1	$\bar{1}$		
	c^{15}								1	1	1		
x^4	c^2		1	2	3	1							
	c^4		$\bar{1}$		2	1	$\bar{1}$	$\bar{2}$	$\bar{1}$				
	c^6		$\bar{1}$	$\bar{2}$	$\bar{4}$	$\bar{4}$	$\bar{4}$	$\bar{3}$	$\bar{1}$				
	c^8			$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{5}$	$\bar{2}$	1	2	1		
	c^{10}					$\bar{1}$	$\bar{1}$	1	1	1			
	c^{12}					1	3	4	4	2			
	c^{14}						1	2	3	2	1	1	1
x^5	c^1		1	1									
	c^3				$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$					
	c^5		$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{2}$	$\bar{1}$			
	c^7			$\bar{2}$	$\bar{4}$	$\bar{5}$	$\bar{4}$	$\bar{2}$	$\bar{2}$	$\bar{1}$			
	c^9					$\bar{1}$	$\bar{1}$	$\bar{1}$	1	2	2	1	
	c^{11}					1	2	2	4	5	4	2	
	c^{13}						1	3	4	4	4	1	
	c^{15}							1	1	1	1		
	c^{17}										$\bar{1}$		

		d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}	d^{11}	d^{12}
x^9	c^1			$\bar{1}$									
	c^5			1	2	2		$\bar{1}$	$\bar{1}$				
	c^7				2	3	2	1					
	c^9					1	2	3	3	3	1		
	c^{13}							$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{4}$	$\bar{2}$	
	c^{15}								$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$	
x^8	c^2			$\bar{1}$	$\bar{1}$	$\bar{1}$							
	c^4			1	$\bar{1}$	$\bar{2}$	$\bar{1}$	$\bar{1}$					
	c^6			1	1	1		$\bar{1}$	$\bar{1}$	$\bar{1}$			
	c^8				1	3	3	4	2				
	c^{10}					1	3	3	3	4	2	1	
	c^{12}							$\bar{2}$	$\bar{3}$	$\bar{2}$		1	
	c^{14}							$\bar{1}$	$\bar{3}$	$\bar{5}$	$\bar{3}$	$\bar{1}$	$\bar{1}$
	c^{16}									$\bar{1}$	$\bar{1}$	$\bar{1}$	
x^7	c^3	$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{2}$	$\bar{1}$					
	c^5				$\bar{2}$	$\bar{4}$	$\bar{4}$	$\bar{3}$	$\bar{1}$				
	c^7				$\bar{1}$	$\bar{1}$	$\bar{1}$	1	1				
	c^9			$\bar{1}$	$\bar{2}$	$\bar{1}$	2	5	5	3	1		
	c^{11}					1	3	4	4	4	2	1	
	c^{13}					1	2	1	$\bar{1}$	$\bar{2}$		1	
	c^{15}								$\bar{1}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	
x^6	c^0			1									
	c^2			$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$						
	c^4		$\bar{1}$	$\bar{4}$	$\bar{4}$	$\bar{4}$	$\bar{3}$	$\bar{1}$					
	c^6		$\bar{2}$	$\bar{4}$	$\bar{5}$	$\bar{4}$	$\bar{2}$	$\bar{2}$	$\bar{1}$				
	c^8		$\bar{1}$	$\bar{2}$	$\bar{2}$	$\bar{1}$	1	1	1				
	c^{10}				1	2	2	4	5	4	2		
	c^{12}				1	2	2	3	4	5	3	1	
	c^{14}						1	1	1	1			
	c^{16}										$\bar{1}$	$\bar{1}$	

*Table of Groundforms.**

Order in the Variables.	Deg. in coeff's of cubic.	Deg. in coeff's of quartic. 0	1	2	3	4	5
0	0			1	1		
	2				1		
	4	1	1	2	3	2	1
	6			1	3	2	1
1	1		1	1			
	3		2	3	2	1	
	5		1	2	2		

Order in the Variables.	Deg. in coeff's of cubic.	Deg. in coeff's of quartic. 0	1	2	3
2	2	1	2	2	1
	4		2	2	
3	1	1	1	1	1
	3	1	1	1	1
4	0		1	1	
	2		1	1	1
5	1		1	1	
6	0				1

System of Two Quartics.

G. F. for differentiants,

Denominator: $(1-d)(1-d^2)^2(1-d^3)(1-\delta)(1-\delta^2)^2(1-\delta^3)(1-d\delta)(1-d^2\delta)$
$(1-d\delta^2)$.

Numerator: $1+d^3+(3d+3d^2-d^4-d^5)\,\delta+(3d+4d^2-d^3-3d^4-2d^5-d^6)\,\delta^2$
$+(1-d^2-2d^4-3d^5-d^6)\,\delta^3+(-d-3d^2-2d^3-d^5+d^7)\,\delta^4$
$+(-d-2d^2-3d^3-d^4+4d^5+3d^6)\,\delta^5+(-d^2-d^3+3d^5+3d^6)\delta^6$
$+(d^4+d^7)\,\delta^7$.

G. F. for covariants, reduced form,

Denominator: $(1-d^2)(1-d^3)(1-\delta^2)(1-\delta^3)(1-d\delta)(1-d^2\delta)(1-d\delta^2)$
$(1-dx^2)(1-dx^4)(1-\delta x^2)(1-\delta x^4)$.

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* The form of ord. 1, deg. 5, 4, and the two forms of ord. 2, deg. 4, 3, given by Gundelfinger, do not appear in this table, and it has been proved by the author that no fundamental forms of either of these types exist.

Numerator:

		δ^0	δ^1	δ^2	δ^3	δ^4	δ^5
x^0	d^0	1					
	d^2			1			
	d^4					1	
x^2	d^0		$\bar{1}$				
	d^1	$\bar{1}$	1	1	1		
	d^2		1	1			
	d^3		1		1		
	d^4						$\bar{1}$
	d^5					$\bar{1}$	
x^4	d^0			1			
	d^1		2		$\bar{1}$	$\bar{1}$	
	d^2	1		$\bar{1}$	$\bar{2}$		
	d^3		$\bar{1}$	$\bar{2}$			
	d^4		$\bar{1}$				$\bar{1}$
	d^5					$\bar{1}$	1

		δ^1	δ^2	δ^3	δ^4	δ^5	δ^6
x^{10}	d^2		1				
	d^4				1		
	d^6						1
x^8	d^1		$\bar{1}$				
	d^2	$\bar{1}$					
	d^3			1		1	
	d^4				1	1	
	d^5			1	1	1	$\bar{1}$
	d^6					$\bar{1}$	
x^6	d^1	1	$\bar{1}$				
	d^2	$\bar{1}$				$\bar{1}$	
	d^3				$\bar{2}$	$\bar{1}$	
	d^4			$\bar{2}$	$\bar{1}$		1
	d^5		$\bar{1}$	$\bar{1}$		2	
	d^6				1		

G. F. for covariants, representative form,

Denominator: $(1-d^2)(1-d^3)(1-\delta^2)(1-\delta^3)(1-d\delta)(1-d^2\delta)(1-d\delta^2)$
$(1-dx^4)(1-d^2x^4)(1-\delta x^4)(1-\delta^2x^4)$.

Numerator:

		δ^0	δ^1	δ^2	δ^3	δ^4	δ^5	δ^6
x^0	d^0	1						
	d^2			1				
	d^4					1		
x^2	d^1		1	1	1			
	d^2		1	1	1			
	d^3		1	1	1			
x^4	d^1		1	1				
	d^2		1	1				
	d^3				1	1		
	d^4				1		$\bar{1}$	$\bar{1}$
	d^5					$\bar{1}$		
	d^6					$\bar{1}$		
x^6	d^0				1			
	d^1		1	1		$\bar{1}$	$\bar{1}$	
	d^2		1	1	$\bar{1}$	$\bar{2}$	$\bar{1}$	
	d^3	1		$\bar{1}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	
	d^4		$\bar{1}$	$\bar{2}$	$\bar{2}$			
	d^5		$\bar{1}$	$\bar{1}$	$\bar{1}$			

		δ^1	δ^2	δ^3	δ^4	δ^5	δ^6	δ^7
x^{14}	d^3			1				
	d^5					1		
	d^7							1
x^{12}	d^4				1	1	1	
	d^5				1	1	1	
	d^6				1	1	1	
x^{10}	d^1			$\bar{1}$				
	d^2			$\bar{1}$				
	d^3	$\bar{1}$	$\bar{1}$		1			
	d^4			1	1			
	d^5					1	1	
	d^6					1	1	
x^8	d^2				$\bar{1}$	$\bar{1}$	$\bar{1}$	
	d^3				$\bar{2}$	$\bar{2}$	$\bar{1}$	
	d^4		$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{1}$		1
	d^5		$\bar{1}$	$\bar{2}$	$\bar{1}$	1	1	
	d^6		$\bar{1}$	$\bar{1}$		1	1	
	d^7				1			

*Table of Groundforms.**

Order in the Variables.	Deg. in coeff's of 2d quartic.	Deg. in coeff's of 1st quartic. 0	1	2	3
0	0			1	1
	1		1	1	
	2	1	1	1	
	3	1			
2	1		1	1	1
	2		1	1	1
	3		1	1	

Order in the Variables.	Deg. in coeff's of 2d quartic.	Deg. in coeff's of 1st quartic. 0	1	2	3
4	0		1	1	
	1	1	1	1	
	2	1	1		
6	0				1
	1		1	1	
	2		1		
	3	1			

The following table exhibits the total numbers of groundforms; the quantics themselves and the absolute constant are included in the numbers.†

	Order of Quantic. 0	1	2	3	4
Order of Quantic. 0	1	2	3	5	6
1		4	6	14	21
2			7	16	19
3				27	62
4					29

* The forms of ord. 4, deg. 2, 2, and of ord. 6, deg. 2, 2, given by Gordan, do not appear in this table, and have been proved by the author to be compound forms.

† Some remarks on the preceding tables (to save delay in going to press) have been made the subject of a separate article in this number.